Representation and Reasoning with Uncertain Temporal Relations

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Abstract

Representation and reasoning with uncertain relations between temporal points is the main goal of this paper. Often humans have to deal with uncertain knowledge. Basically uncertainty includes two main aspects: inexactness (probabilistic aspect) and inconsistency. Temporal area is not an exception. In this paper we suggest one way to represent uncertain relations between temporal points. This representation allows to estimate the degree of exactness in temporal relations by providing probabilities for possible temporal relations, and also to derive the structure of possible inconsistent relation. We consider inconsistent relation as conflicting knowledge when describing the same relation.

The basic vector with seven parameters that represents a relation between two temporal points consists of two parts: inexact forth and inconsistent triad. The first part distributes probabilities among basic relations "<", ">", "=" and probability of inconsistent relation. The second part represents the composition for possible inconsistent relation: percentage of "<", of ">" and of "=" within the inconsistent relation. The reasoning mechanism, proposed in this paper, allows to compose, inverse and add such temporal relations by recalculating the values of the resulting vector. Such representation makes it possible to evaluate reasoning result giving exact measures for inexact and inconsistent parts of a resulting relation.

1. Introduction

The problem of representation and reasoning with temporal knowledge arises in a wide range of disciplines, including computer science, philosophy, psychology, and linguistics. In computer science, it is a core problem of information systems, program verification, artificial intelligence, and other areas involving modeling process. During the early 80s some general pieces of work aimed at providing general theories of time and action appeared, such as McDermott's temporal logic [2], Allen's theory of action and time [1], Vilain's theory of time [5].

These proposals were good to establish the two main contenders as temporal ontological primitives (point and interval), to make initial proposals on representational issues and reasoning algorithms, to point out the general problems (the reasoning by default, the interaction of actions, the use of a temporal reasoner in application) and for showing that a more basic machinery has to be build before defining a general theory of time [6]. In this paper we consider relations between temporal points, and we take temporal points as ontological primitives.

Often humans have to deal with uncertainty, which includes two main aspects: inexactness (probabilistic) and inconsistency. Temporal area is not an exception. In this paper we suggest one way to represent uncertain temporal relations between points. This representation allows to estimate the degree of exactness in temporal relations and also to derive the structure of possible inconsistent relation. The proposed approach can be considered consisting of two parts: first one defines the representation model, and the second part deals with reasoning mechanism which uses the proposed representation model.

Approaches to temporal reasoning deal with inexact temporal knowledge in the following way. It is supposed that inexact temporal relation is a disjunction of two or more exact relations. If temporal information is inexact in some applications, these approaches process it without consideration of the probabilities of each of basic relations in inexact one. This seems to be a weakness if we are speaking about decision support systems or systems where the probability of each alternative plays an important role. The really important task that arises in such systems is not only to predict the result relation, but also to provide probabilities for each of alternatives. Hence one of the goals we have stated to be achieved in proposed representation model is to include the ability to provide probabilities for possible temporal relations.

In many situations there is a need to reason with inconsistent knowledge [3]. These inconsistencies may occur, for example, due to sources of information that are not fully reliable and thus information contains contradiction. Multiple experts opinions is such situation when inconsistency may occur. We consider inconsistent relation as conflicting knowledge when describing the same relation. It might be so that such knowledge was obtained from several knowledge sources. The widely used definition of consistency is the following: the *consistent* knowledge supposes absence of contradiction and *inconsistent* knowledge contains contradiction [3]. One can ask why it is necessary to formalize inconsistency, it simply determines some kind of error and we should think how to avoid it, but not to define. Inconsistency surrounds us everywhere. Inconsistency in information is the norm, and we should feel happy to be able to formalize it [7]. In all areas of human behavior one have to resolve inconsistencies that occur very often. But people usually don’t even notice that the information they have is inconsistent. They just use it and apply human reasoning mechanism for making decision. The difference between artificial and real (human) intelligence behavior when an inconsistency occurs, concerns interpretation of it. To a human, resolving inconsistencies is not necessarily done by “restoring” consistency but by supplying rules telling one how to act when the inconsistency arises [7]. For artificial intelligence there is an urgent need to revise the view that inconsistency is a “bad” thing, and instead view it as mostly a “good” thing.

We often have to deal with inconsistent temporal information when we make decision about temporal relations, when we deal with amalgamation of temporal databases, planning under uncertainty, interpretation of natural language and so on. But the logic for dealing with occurred inconsistency almost always use classical logic approach, which is aimed directly for restoring consistency. Restoring consistency often means elimination of a source of inconsistency on global or local level. The published approaches are aimed to find consistency consider only consistent part of knowledge, and this means loss of information from inconsistent parts. We argue that in many application areas, for example decision making systems it is essential to have complete information about relation even if it contains contradiction. Any loss of information may cause deriving incorrect knowledge and hence to wrong decision. This point of view is central in our consideration. In this paper we propose mechanism for representing uncertain temporal relations, which includes ability to represent both inexact and inconsistent relations, and reasoning with them.

2. Representation of Uncertain Relations

This section deals with representation model for uncertain relations between temporal points. First we will give some definitions, that serve as a background for basic concepts used throughout the paper. And we start with the definition of temporal relations we are dealing with.

Definition 2.1. *Basic relations* that can hold between temporal points are “<”, “=” and “>”. We will call them *exact* temporal relations between points. Possible disjunction of these relations, namely, “≤” (< or =), “≥” (> or =), “≠” (< or >) and “?” (< or = or >) we will call *inexact* temporal relations between points.

So there are 3 exact relations and 4 inexact relations. That was usual consideration in the published literature about temporal relations. But we should evaluate this definition from the perspective of uncertainty representation. What kind of uncertain relations can be derived at all, and does this definition is able to formalize them. Let us remind that we consider uncertainty can be obtained by two ways: inexactness in defining information, and inconsistency. The Definition 2.1 is able to specify all the inexact relations between two temporal points, but is does not have any idea how to formalize inconsistency. Moreover, it was not supposed at all to do this in published approaches in this area. In this paper we interpret an inconsistent temporal relation in the following way.

Definition 2.2. *Inconsistent temporal relation* is conjunction of two or more basic temporal relations, and it inherits all the temporal meanings of the basic temporal relations included.

Inconsistent relation includes conflicting meanings of information when describing the same relation. For example, if the one expert says: “This relation is “<” and the another one says: “This relation is “>”. The common opinion is the relation “< and >” and it is denoted as inconsistent relation “< and >”. In other words, we assume that if we are given contradict information about the same relation, we will define this relation using all the given information. It is supposed to store the inconsistent knowledge, but not to try to restore consistency. By this we are going to distinguish between inconsistent relations, taking into account how they were obtained. This can be achieved by supposing that each inconsistent relation has it’s own structure. This structure is defined consisting of basic relations, that have composed the inconsistent relation, as it is shown by the following definition.

Definition 2.3. An inconsistent relation is composed of basic relations, e.g. “<”, “=”, “>”. We define the composition of inconsistent relation as following triad: $[d^<, d^=, d^>]$,

where $d^<$, $d^=$, $d^>$ denote the percentage content for each of basic relations within the relation between two temporal points, and $d^< + d^= + d^> = 1$.

Definition 2.4. Value of *exactness* of any of basic relations between temporal points is the probability that exactly this relation holds between the given two temporal points. Since we have three basic relations (“<”, “>“, “=”) plus possible inconsistent relation between temporal points, we have the following exactness variables:

$e^<, e^=, e^>, e^i$ – values of exactness of relations <, =, >, and inconsistent relation, respectively.

The sum of these variables is equal to 1, since they include probabilities of all possible relations that can hold between the two temporal points, $e^< + e^= + e^> + e^i = 1$.

Definition 2.5. Representation of any relation between two temporal points ***a*** and ***b*** takes into account the appropriate values of exactness and the composition of inconsistency, and it is defined by the following vector:

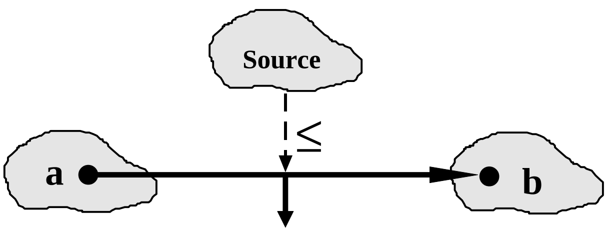
$$\left(e^<, e^=, e^>, e^i \left[d^<, d^=, d^>\right]\right)_{a,b}, \text{ where } e^< + e^= + e^> + e^I = 1 \text{ and } d^< + d^= + d^> = 1,$$

and values $d^<$, $d^=$, $d^>$ are defined only in the case when $e^i \neq 0$.

Definition 2.6. The initial exactness values in the case of “?” relation between two temporal points are equal respectively to $e_0^<$, $e_0^=$, $e_0^>$ and $e_0^< + e_0^= + e_0^> = 1$.

Let us consider three examples that illustrate the usability of proposed representation model.

Example 1. We consider a relation between two temporal points ***a*** and ***b*** (Fig.1).

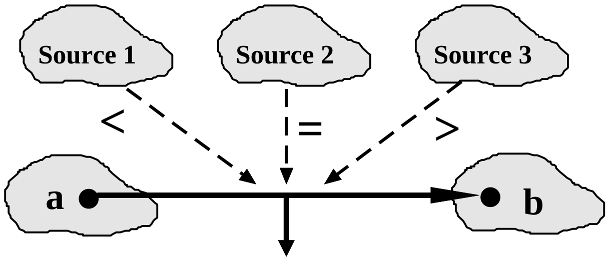


$$L_{a,b} = \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right)$$

Figure 1. Representation of inexact relation

In this example we have only one source of information and the relations that is provided is inexact relation, and hence should be defined through values of exactness of representation vector. The values of inconsistent group are not defined with an accordance to the Definition 2.5, since probability of inconsistent relation e^i is equal to 0. This is example of temporal relation without any inconsistency.

Example 2. The Figure 2 shows the representation of inconsistent temporal relation. Note, that the cause of inconsistency is several sources of information, that give us contradict information.



$$L_{a,b} = \left(0, 0, 0, 1\left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]\right)$$

Figure 2. Representation of inconsistent relation

Here we assume that the initial probabilities $e_0^<$, $e_0^=$, $e_0^>$ are equal to each other, and hence they equal to $\frac{1}{3}$. This impacts the distribution inside inconsistent group. By other values of $e_0^<$, $e_0^=$, $e_0^>$ the values of variables inside inconsistent group will be another.

Example 3. Now let us consider uncertain relation. In Figure 3 two sources of information provide us inexact knowledge.

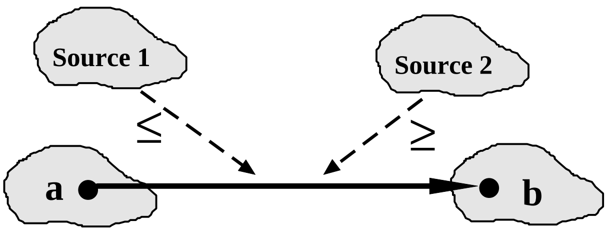


Figure 3. Representation of uncertain relation

First we specify representation vectors for information from "Source 1" and "Source 2". The vectors look like: for "Source 1" - $\left(\frac{1}{2}, \frac{1}{2}, 0, 0\right)$, for "Source 2" - $\left(0, \frac{1}{2}, \frac{1}{2}, 0\right)$. Then we have some-how to combine these vectors to derive the common knowledge. To do this we need to have a reasoning mechanism that would use the representation vectors and consists of three operations: inverse, composition, and addition that are defined under uncertain relations between temporal points. The next section will present such mechanism and the Example 3 will be continued in the end of Section 3.

3. Reasoning with Uncertain Relations

The reasoning mechanism includes three operations: inverse, composition and addition. First two operations are classical for all mechanisms that are intended for performing reasoning under temporal relations. The third "usual" operation that is described in published approaches is intersection. But in the proposed in this paper mechanism we have replaced it with addition operation. Here we have to distinguish between these operations. The difference between them seems to be like the difference between ways to handle inconsistency when it has occurred. The intersection operation is aimed to find out the “common part” in relations to be intersected. This lefts no chances for occurring inconsistency, but descend potential troubles that were mentioned in Section 2. The addition operation is intended for summarizing all the information provided in relations under operation. When contradiction is derived the inconsistent part of representation vector is changed. The proposed in previous section representation model is used in all these operations. Because of complicated proof, we would not provide exhaustive formalisms, but only the necessitate for understanding. The next definition gives us the notion of relation between any temporal points. We will need it further while defining operations for reasoning.

Definition 3.1. Let us suppose that we are given two temporal points ***a*** and ***b***. and we have a relation ***L*** that holds between these points. Then the predicate of truth as follows:

$$P(a,\ L,\ b)=\begin{cases} \mathit{true},\ \text{if relation } L \text{ holds between temporal points } a \text{ and } b; \\ \mathit{false},\ \text{otherwise} \end{cases}$$

Definition 3.2. The inverse operation (Fig.4) is denoted “~” and defined by the following equation:

$$\mathrm{P}(\mathrm{a}, \mathrm{L}_{\mathrm{a,b}}, \mathrm{b}) \Leftrightarrow \mathrm{P}(\mathrm{b}, \tilde{\mathrm{L}}_{\mathrm{a,b}}, \mathrm{a}),$$ where ***a***, ***b*** are temporal points,

$\mathrm{L}_{\mathrm{a,b}}$=$\left(e_1^<, e_1^=, e_1^>, e_1^i[d_1^<, d_1^=, d_1^>]\right)$ is original relation between points ***a*** and ***b***, $\tilde{\mathrm{L}}_{\mathrm{a,b}}$ is the result of inversion represented by the relation $\mathrm{L}_{\mathrm{b,a}}$=$\left(e_r^<, e_r^=, e_r^>, e_r^i[d_r^<, d_r^=, d_r^>]\right)$. Then we suggest the following formulas to recalculate the variables in representation vector:

$e_r^< = e_1^>, e_r^= = e_1^=, e_r^> = e_1^<, d_r^< = d_1^>, d_r^= = d_1^=,\ d_r^> = d_1^<$.

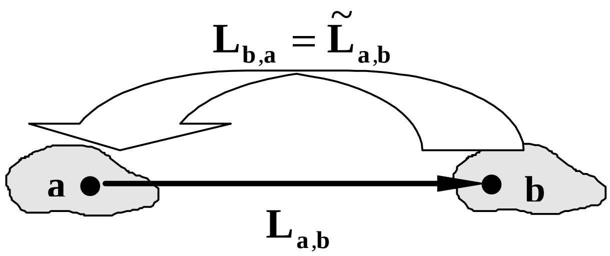


Figure 4. Inverse operation

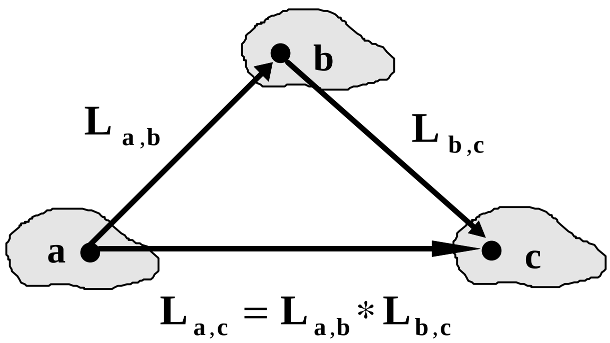


Figure 5. Composition operation

Definition 3.3. The composition operation (Fig.5) is denoted “*” and defined by the following equation:

$$P(a, L_{a,b}, b) \wedge P(b, L_{b,c}, c) \Leftrightarrow P(a, L_{a,b} * L_{b,c}, c),$$

where ***a***, ***b*** and ***c*** are temporal points, $L_{a,b}$=$\left(e_1^<, e_1^=, e_1^>, e_1^i[d_1^<, d_1^=, d_1^>]\right)$ is the first original relation between points ***a*** and ***b***, $L_{b,c}$=$\left(e_2^<, e_2^=, e_2^>, e_2^i[d_2^<, d_2^=, d_2^>]\right)$ is the second original relation between points ***b*** and ***c***, $L_{a,b} * L_{b,c}$ is the result of composition represented by the relation $L_{a,c}$=$\left(e_r^<, e_r^=, e_r^>, e_r^i[d_r^<, d_r^=, d_r^>]\right)$ between points ***a*** and ***c***. Then we suggest the following formulas to recalculate the variables in representation vector:

$$e_r^< = e_1^< \cdot e_2^< + e_1^< \cdot e_2^= + e_1^= \cdot e_2^< + e_0^< \cdot e_1^< \cdot e_2^> + e_0^< \cdot e_1^> \cdot e_2^<, e_r^= = e_1^= \cdot e_2^= + e_0^= \cdot e_1^< \cdot e_2^> + e_0^= \cdot e_1^> \cdot e_2^<,$$
$$e_r^> = e_1^> \cdot e_2^> + e_1^= \cdot e_2^> + e_1^> \cdot e_2^= + e_0^> \cdot e_1^< \cdot e_2^> + e_0^> \cdot e_1^> \cdot e_2^<, e_r^i = \Sigma^< + \Sigma^= + \Sigma^>,$$

$$d_r^{<} = \frac{\Sigma^{<}}{\Sigma^{<} + \Sigma^{=} + \Sigma^{>}},\; d_r^{=} = \frac{\Sigma^{=}}{\Sigma^{<} + \Sigma^{=} + \Sigma^{>}},\; d_r^{>} = \frac{\Sigma^{>}}{\Sigma^{<} + \Sigma^{=} + \Sigma^{>}}, \text{ where}$$

$$\Sigma^{<} = e_1^{<} \cdot e_2^{i} \cdot d_2^{<} + e_2^{<} \cdot e_1^{i} \cdot d_1^{<} + e_1^{<} \cdot e_2^{i} \cdot d_2^{=} + e_1^{=} \cdot e_2^{i} \cdot d_2^{<} + e_2^{<} \cdot e_1^{i} \cdot d_1^{=} + e_2^{=} \cdot e_1^{i} \cdot d_1^{<} + e_1^{<} \cdot e_2^{i} \cdot d_2^{>} \cdot e_0^{<} + e_1^{>} \cdot e_2^{i} \cdot d_2^{<} \cdot e_0^{<} +$$
$$+ e_2^{<} \cdot e_1^{i} \cdot d_1^{>} \cdot e_0^{<} + e_2^{>} \cdot e_1^{i} \cdot d_1^{<} \cdot e_0^{<} + e_1^{i} \cdot d_1^{<} \cdot e_2^{i} \cdot d_2^{<} + e_1^{i} \cdot d_1^{<} \cdot e_2^{i} \cdot d_2^{=} + e_1^{i} \cdot d_1^{<} \cdot e_2^{i} \cdot d_2^{>} \cdot e_0^{<} + e_1^{i} \cdot d_1^{=} \cdot e_2^{i} \cdot d_2^{<} + e_1^{i} \cdot d_1^{>} \cdot e_2^{i} \cdot d_2^{<} \cdot e_0^{<}.$$

$$\Sigma^{=} = e_1^{=} \cdot e_2^{i} \cdot d_2^{=} + e_2^{=} \cdot e_1^{i} \cdot d_1^{=} + e_1^{<} \cdot e_2^{i} \cdot d_2^{>} \cdot e_0^{=} + e_1^{>} \cdot e_2^{i} \cdot d_2^{<} \cdot e_0^{=} + e_2^{<} \cdot e_1^{i} \cdot d_1^{>} \cdot e_0^{=} + e_2^{>} \cdot e_1^{i} \cdot d_1^{<} \cdot e_0^{=} +$$
$$e_1^{i} \cdot d_1^{<} \cdot e_2^{i} \cdot d_2^{>} \cdot e_0^{=} + e_1^{i} \cdot d_1^{<} \cdot e_2^{i} \cdot d_2^{>} \cdot e_0^{=}.$$

$$\Sigma^{>} = e_1^{>} \cdot e_2^{i} \cdot d_2^{>} + e_2^{>} \cdot e_1^{i} \cdot d_1^{>} + e_1^{<} \cdot e_2^{i} \cdot d_2^{>} \cdot e_0^{>} + e_1^{=} \cdot e_2^{i} \cdot d_2^{>} + e_1^{>} \cdot e_2^{i} \cdot d_2^{<} \cdot e_0^{>} + e_1^{>} \cdot e_2^{i} \cdot d_2^{=} + e_2^{<} \cdot e_1^{i} \cdot d_1^{>} \cdot e_0^{>} + e_2^{=} \cdot e_1^{i} \cdot d_1^{>} +$$
$$+ e_2^{>} \cdot e_1^{i} \cdot d_1^{<} \cdot e_0^{>} + e_2^{>} \cdot e_1^{i} \cdot d_1^{=} + e_1^{i} \cdot d_1^{>} \cdot e_2^{i} \cdot d_2^{>} + e_1^{i} \cdot d_1^{>} \cdot e_2^{i} \cdot d_2^{=} + e_1^{i} \cdot d_1^{<} \cdot e_2^{i} \cdot d_2^{>} \cdot e_0^{>} + e_1^{i} \cdot d_1^{=} \cdot e_2^{i} \cdot d_2^{>} + e_1^{i} \cdot d_1^{>} \cdot e_2^{i} \cdot d_2^{<} \cdot e_0^{>}.$$

These formulas were obtained by finding all possible compositions between all elements taken from both vectors. We use the composition table for temporal points that had been proposed by Vilain and Kautz in [4] (Fig. 6).

*	=	<	≤	>	≥	≠
=	=	<	≤	>	≥	≠
<	<	<	<	?	?	?
≤	≤	<	≤	?	?	?
>	>	?	?	>	>	?
≥	≥	?	?	>	≥	?
≠	≠	?	?	?	?	?

Figure 6. Composition table

∩	=	<	≤	>	≥	≠
=	=	∅	=	∅	=	∅
<	∅	<	<	∅	∅	<
≤	=	<	≤	∅	=	<
>	∅	∅	∅	>	>	>
≥	=	∅	=	>	≥	>
≠	∅	<	<	>	>	≠

Figure 7. Intersection table

Using this table, we see the following cases support this probability: (1) "<"*"<"; (2) "<"*"="; (3) "="*"<", and also (4) "<"*">", and (5) ">"*"<". Probabilities of the first three cases fully support the $e_r^{<}$. The value of $e_0^{<}$ defines the parts of support for cases (4) and (5) which are belong to the support of $e_r^{<}$. Then it follows: $e_r^{<} = \underbrace{e_1^{<} \cdot e_2^{<}}_{case\ 1} + \underbrace{e_1^{<} \cdot e_2^{=}}_{case\ 2} + \underbrace{e_1^{=} \cdot e_2^{<}}_{case\ 3} + \underbrace{e_0^{<} \cdot e_1^{<} \cdot e_2^{>}}_{case\ 4} + \underbrace{e_0^{<} \cdot e_1^{>} \cdot e_2^{<}}_{case\ 5}$. Similarly:

$$e_r^{=} = e_1^{=} \cdot e_2^{=} + e_0^{=} \cdot e_1^{<} \cdot e_2^{>} + e_0^{=} \cdot e_1^{>} \cdot e_2^{<},\; e_r^{>} = e_1^{>} \cdot e_2^{>} + e_1^{=} \cdot e_2^{>} + e_1^{>} \cdot e_2^{=} + e_0^{>} \cdot e_1^{<} \cdot e_2^{>} + e_0^{>} \cdot e_1^{>} \cdot e_2^{<}.$$

According to the Definition 2.3 $d_r^{<}$ is the percentage value of the "<" relation within the inconsistent one. Based on composition table it follows that the following cases support this percentage value:

(1) "<" (in. group 1)* "<" (inc. group 2);
(2) "<" (in. group 1)* "=" (inc. group 2);
(3) "=" (in. group 1)*"<"(inc. group 2);
(4) "<" (in. group 1)* ">" (inc. group 2);
(5)">" (in. group 1)*"<" (inc. group 2);
(6) "<" (inc. group 1)*"<" (in. group 2);
(7)"<" (inc. group 1)*"=" (in. group 2);
(8) "=" (inc. group 1)*"<" (in. group 2);
(9) "<" (inc. group 1) * ">" (in. group 2);
(10)">" (inc. group 1)*"<" (in. group 2);
(11) "<" (inc. group 1)*"<" (inc. group 2);
(12)"<" (inc. group 1)*"=" (inc. group 2);
(13) "=" (inc. group 1)* "<" (inc. group 2);
(14)"<" (inc. group 1)*">" (inc. group 2);
(15) ">" (inconsistent group 1)* "<" (inconsistent group 2),

where inc. group 1 is $[d_1^{<}, d_1^{=}, d_1^{>}]$, in. group 1 is $\left(e_1^{<}, e_1^{=}, e_1^{>}, e_1^{i}\right)$, inc. group 2 is $[d_2^{<}, d_2^{=}, d_2^{>}]$, and in. group 2 is $\left(e_2^{<}, e_2^{=}, e_2^{>}, e_2^{i}\right)$. In cases 4,5,9,10,14,15 we use partial support defined by the probability $e_0^{<}$. Thus the value of support $\Sigma^{<}$ for the resulting value $d_r^{<}$ is calculated as the sum of all cases.

Similarly we obtain expressions for $\Sigma^{=}$ and $\Sigma^{>}$. The total support of inconsistent relation e_r^{i} is equal to $\Sigma^{<} + \Sigma^{=} + \Sigma^{>}$ because it based on probabilities for all cases of inconsistency.

Final values for $d_r^<, d_r^=, d_r^>$ are calculated using the above support values towards satisfying the requirement $d_r^< + d_r^= + d_r^> = 1$ as follows:

$$d_r^< = \frac{\Sigma^<}{\Sigma^< + \Sigma^= + \Sigma^>}, \quad d_r^= = \frac{\Sigma^=}{\Sigma^< + \Sigma^= + \Sigma^>}, \; d_r^> = \frac{\Sigma^>}{\Sigma^< + \Sigma^= + \Sigma^>}.$$

Definition 3.4. The addition operation (Fig.8) is denoted "+" and defined by the following equation:

$$P(a, L_1, b) \wedge P(a, L_2, b) \Leftrightarrow P(a, L_1 + L_2, b),$$

where ***a***, ***b*** are temporal points, $L_1 = \left(e_1^<, e_1^=, e_1^>, e_1^i[d_1^<, d_1^=, d_1^>]\right)$ is the first original relation between points ***a*** and ***b***, $L_2 = \left(e_2^<, e_2^=, e_2^>, e_2^i[d_2^<, d_2^=, d_2^>]\right)$ is the second original relation between points ***a*** and ***b***, $L_1 + L_2$ is the result of addition represented by the relation $L_{a,b} = \left(e_r^<, e_r^=, e_r^>, e_r^i[d_r^<, d_r^=, d_r^>]\right)$.

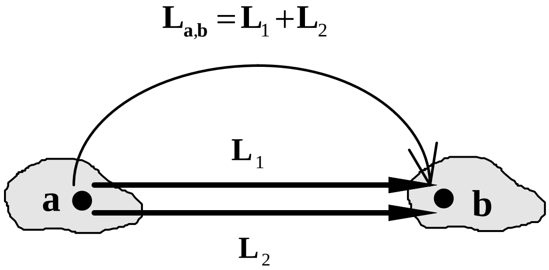


Figure 8. Addition operation for relations between temporal points

Then we suggest the following formulas to recalculate the variables in representation vector:

$$e_r^< = e_1^< \cdot e_2^<, e_r^= = e_1^= \cdot e_2^=, e_r^> = e_1^> \cdot e_2^>, e_r^i = \Sigma^< + \Sigma^= + \Sigma^>,$$

$$d_r^< = \frac{\Sigma^<}{\Sigma^< + \Sigma^= + \Sigma^>}, d_r^= = \frac{\Sigma^=}{\Sigma^< + \Sigma^= + \Sigma^>}, \; d_r^> = \frac{\Sigma^>}{\Sigma^< + \Sigma^= + \Sigma^>}, \text{ where}$$

$$\Sigma^< = e_1^< \cdot e_2^= \cdot \frac{e_1^<}{e_1^< + e_2^=} + e_1^< \cdot e_2^> \cdot \frac{e_1^<}{e_1^< + e_2^>} + e_1^= \cdot e_2^< \cdot \frac{e_2^<}{e_1^= + e_2^<} + e_1^> \cdot e_2^< \cdot \frac{e_2^<}{e_1^> + e_2^<} + e_1^< \cdot e_2^i \cdot d_2^< +$$

$$+ e_2^< \cdot e_1^i \cdot d_1^< + e_1^< \cdot e_2^i \cdot e_2^= \cdot \frac{e_1^<}{e_1^< + e_2^i \cdot e_2^=} + e_1^< \cdot e_2^i \cdot e_2^> \cdot \frac{e_1^<}{e_1^< + e_2^i \cdot e_2^>} + e_1^= \cdot e_2^i \cdot d_2^< \cdot \frac{e_2^i \cdot d_2^<}{e_1^= + e_2^i \cdot d_2^<} +$$

$$+ e_2^< \cdot e_1^i \cdot e_1^= \cdot \frac{e_2^<}{e_2^< + e_1^i \cdot e_1^=} + e_2^< \cdot e_1^i \cdot e_1^> \cdot \frac{e_2^<}{e_2^< + e_1^i \cdot e_1^>} + e_2^= \cdot e_1^i \cdot d_1^< \cdot \frac{e_1^i \cdot d_1^<}{e_2^= + e_1^i \cdot d_1^<} +$$

$$+ e_2^> \cdot e_1^i \cdot d_1^< \cdot \frac{e_1^i \cdot d_1^<}{e_2^> + e_1^i \cdot d_1^<} + e_1^i \cdot d_1^< \cdot e_2^i \cdot d_2^< + e_1^i \cdot d_1^< \cdot e_2^i \cdot d_2^= \cdot \frac{e_1^i \cdot d_1^<}{e_1^i \cdot d_1^< + e_2^i \cdot d_2^=} + e_1^i \cdot d_1^< \cdot e_2^i \cdot d_2^> \cdot \frac{e_1^i \cdot d_1^<}{e_1^i \cdot d_1^< + e_2^i \cdot d_2^>} +$$

$$+ e_1^i \cdot d_1^= \cdot e_2^i \cdot d_2^< \cdot \frac{e_2^i \cdot d_2^<}{e_1^i \cdot d_1^= + e_2^i \cdot d_2^<} + e_1^i \cdot d_1^> \cdot e_2^i \cdot d_2^< \cdot \frac{e_2^i \cdot d_2^<}{e_1^i \cdot d_1^> + e_2^i \cdot d_2^<},$$

$$\Sigma^= = e_1^< \cdot e_2^= \cdot \frac{e_2^=}{e_1^< + e_2^=} + e_1^= \cdot e_2^< \cdot \frac{e_1^=}{e_1^= + e_2^<} + e_1^= \cdot e_2^> \cdot \frac{e_1^=}{e_1^= + e_2^>} + e_1^> \cdot e_2^= \cdot \frac{e_2^=}{e_1^> + e_2^=} +$$

$$+ e_1^= \cdot e_2^i \cdot d_2^= + e_2^= \cdot e_1^i \cdot d_1^= + e_1^< \cdot e_2^i \cdot d_2^= \cdot \frac{e_2^i \cdot d_2^=}{e_1^< + e_2^i \cdot d_2^=} + e_1^= \cdot e_2^i \cdot d_2^< \cdot \frac{e_1^=}{e_1^= + e_2^i \cdot d_2^<} + e_1^= \cdot e_2^i \cdot d_2^> \cdot \frac{e_1^=}{e_1^= + e_2^i \cdot d_2^>} +$$

$$+e_1^{>}\cdot e_2^{i}\cdot d_2^{=}\cdot\frac{e_2^{i}\cdot d_2^{=}}{e_1^{>}+e_2^{i}\cdot d_2^{=}}+e_2^{<}\cdot e_1^{i}\cdot d_1^{=}\cdot\frac{e_1^{i}\cdot d_1^{=}}{e_2^{<}+e_1^{i}\cdot d_1^{=}}+e_2^{=}\cdot e_1^{i}\cdot d_1^{<}\cdot\frac{e_2^{=}}{e_2^{=}+e_1^{i}\cdot d_1^{<}}+e_2^{=}\cdot e_1^{i}\cdot d_1^{>}\cdot\frac{e_2^{=}}{e_2^{=}+e_1^{i}\cdot d_1^{>}}+$$

$$+e_2^{>}\cdot e_1^{i}\cdot d_1^{=}\cdot\frac{e_1^{i}\cdot d_1^{=}}{e_2^{>}+e_1^{i}\cdot d_1^{=}}+e_1^{i}\cdot d_1^{=}\cdot e_2^{i}\cdot d_2^{=}+e_1^{i}\cdot d_1^{<}\cdot e_2^{i}\cdot d_2^{=}\cdot\frac{e_2^{i}\cdot d_2^{=}}{e_1^{i}\cdot d_1^{<}+e_2^{i}\cdot d_2^{=}}+e_1^{i}\cdot d_1^{=}\cdot e_2^{i}\cdot d_2^{<}\cdot\frac{e_1^{i}\cdot d_1^{=}}{e_1^{i}\cdot d_1^{=}+e_2^{i}\cdot d_2^{<}}+$$

$$+e_1^{i}\cdot d_1^{=}\cdot e_2^{i}\cdot d_2^{>}\cdot\frac{e_1^{i}\cdot d_1^{=}}{e_1^{i}\cdot d_1^{=}+e_2^{i}\cdot d_2^{>}}+e_1^{i}\cdot d_1^{>}\cdot e_2^{i}\cdot d_2^{=}\cdot\frac{e_2^{i}\cdot d_2^{=}}{e_1^{i}\cdot d_1^{>}+e_2^{i}\cdot d_2^{=}},$$

$$\Sigma^{>}=e_1^{<}\cdot e_2^{>}\cdot\frac{e_2^{>}}{e_1^{<}+e_2^{>}}+e_1^{=}\cdot e_2^{>}\cdot\frac{e_2^{>}}{e_1^{=}+e_2^{>}}+e_1^{>}\cdot e_2^{<}\cdot\frac{e_1^{>}}{e_1^{>}+e_2^{<}}+e_1^{>}\cdot e_2^{=}\cdot\frac{e_1^{>}}{e_1^{>}+e_2^{=}}+e_1^{>}\cdot e_2^{i}\cdot d_2^{>}+$$

$$+e_2^{>}\cdot e_1^{i}\cdot d_1^{>}+e_1^{<}\cdot e_2^{i}\cdot d_2^{>}\cdot\frac{e_2^{i}\cdot d_2^{>}}{e_1^{<}+e_2^{i}\cdot d_2^{>}}+e_1^{=}\cdot e_2^{i}\cdot d_2^{>}\cdot\frac{e_2^{i}\cdot d_2^{>}}{e_1^{=}+e_2^{i}\cdot d_2^{>}}+e_1^{>}\cdot e_2^{i}\cdot d_2^{<}\cdot\frac{e_1^{>}}{e_1^{>}+e_2^{i}\cdot d_2^{<}}+$$

$$+e_2^{<}\cdot e_1^{i}\cdot d_1^{>}\cdot\frac{e_1^{i}\cdot d_1^{>}}{e_2^{<}+e_1^{i}\cdot d_1^{>}}+e_2^{=}\cdot e_1^{i}\cdot d_1^{>}\cdot\frac{e_1^{i}\cdot d_1^{>}}{e_2^{=}+e_1^{i}\cdot d_1^{>}}+e_2^{>}\cdot e_1^{i}\cdot d_1^{<}\cdot\frac{e_2^{>}}{e_2^{>}+e_1^{i}\cdot d_1^{<}}+$$

$$+e_2^{>}\cdot e_1^{i}\cdot d_1^{=}\cdot\frac{e_2^{>}}{e_2^{>}+e_1^{i}\cdot d_1^{=}}+e_1^{i}\cdot d_1^{>}\cdot e_2^{i}\cdot d_2^{>}+e_1^{i}\cdot d_1^{<}\cdot e_2^{i}\cdot d_2^{>}\cdot\frac{e_2^{i}\cdot d_2^{>}}{e_1^{i}\cdot d_1^{<}+e_2^{i}\cdot d_2^{>}}+e_1^{i}\cdot d_1^{=}\cdot e_2^{i}\cdot d_2^{>}\cdot\frac{e_2^{i}\cdot d_2^{>}}{e_1^{i}\cdot d_1^{=}+e_2^{i}\cdot d_2^{>}}+$$

$$+e_1^{i}\cdot d_1^{>}\cdot e_2^{i}\cdot d_2^{<}\cdot\frac{e_2^{i}\cdot d_1^{>}}{e_1^{i}\cdot d_1^{>}+e_2^{i}\cdot d_2^{<}}+e_1^{i}\cdot d_1^{>}\cdot e_2^{i}\cdot d_2^{=}\cdot\frac{e_1^{i}\cdot d_1^{>}}{e_1^{i}\cdot d_1^{>}+e_2^{i}\cdot d_2^{=}}.$$

To obtain addition result we need to consider all possible combinations between all elements taken from both vectors and decide which resulting value should be supported by the probability of every combination. According to the Definition 2.4, the value $e_r^{<}$ is the probability that exactly the relation "<" holds between temporal points ***a*** and ***b***. According to intersection table (Fig.7), the only case supporting this probability is: "<"+"<". Thus $e_r^{<}$ is calculated as follows: $e_r^{<}=e_1^{<}\cdot e_2^{<}$. Similarly $e_r^{=}=e_1^{=}\cdot e_2^{=}$, $e_r^{>}=e_1^{>}\cdot e_2^{>}$.

One can see that the only cases supporting inexact group require the same operands taken from both original vectors. According to the Definition 2.2 of inconsistent relation, all other cases give support to inconsistent group of resulting relation.

According to the Definition 2.3, $d_r^{<}$ is the percentage value of the "<" relation within the inconsistent group of the relation L_r between temporal points ***a*** and ***b***. Based on intersection table it follows that the following cases support this percentage value:

(1)	"<" (inexact group 1)	+	"=" (inexact group 2);
(2)	"=" (inexact group 1)	+	"<" (inexact group 2);
(3)	"<" (inexact group 1)	+	">" (inexact group 2);
(4)	">" (inexact group 1)	+	"<" (inexact group 2);
(5)	"<" (inexact group 1)	+	"<" (inconsistent group 2);
(6)	"<" (inexact group 1)	+	"=" (inconsistent group 2);
(7)	"=" (inexact group 1)	+	"<" (inconsistent group 2);
(8)	"<" (inexact group 1)	+	">" (inconsistent group 2);
(9)	">" (inexact group 1)	+	"<" (inconsistent group 2);
(10)	"<" (inconsistent group 1)	+	"<" (inexact group 2);
(11)	"<" (inconsistent group 1)	+	"=" (inexact group 2);
(12)	"=" (inconsistent group 1)	+	"<" (inexact group 2);
(13)	"<" (inconsistent group 1)	+	">" (inexact group 2);
(14)	">" (inconsistent group 1)	+	"<" (inexact group 2);

(15) “<” (inconsistent group 1) + “<” (inconsistent group 2);
(16) “<” (inconsistent group 1) + “=” (inconsistent group 2);
(17) “=” (inconsistent group 1) + “<” (inconsistent group 2);
(18) “<” (inconsistent group 1) + “>” (inconsistent group 2);
(19) “>” (inconsistent group 1) + “<” (inconsistent group 2),

where inconsistent group 1 is $[d_1^<, d_1^=, d_1^>]$, inexact group 1 is $\left(e_1^<, e_1^=, e_1^>, e_1^i\right)$, inconsistent group 2 is $[d_2^<, d_2^=, d_2^>]$, and inexact group 2 is $\left(e_2^<, e_2^=, e_2^>, e_2^i\right)$. In cases 5,10,15 we use full support of the case to the resulting value. In other cases the divide the support in proportion between the probabilities of appropriate values. Thus the value of support $\Sigma^<$ for the resulting value $d_r^<$ is calculated as follows:

$$
\begin{aligned}
\Sigma^< = {} & \underbrace{\frac{e_1^<}{e_1^< + e_2^=} \cdot e_1^< \cdot e_2^=}_{case\ 1} + \underbrace{\frac{e_2^<}{e_1^= + e_2^<} \cdot e_1^= \cdot e_2^<}_{case\ 2} + \underbrace{\frac{e_1^<}{e_1^< + e_2^>} \cdot e_1^< \cdot e_2^>}_{case\ 3} + \underbrace{\frac{e_2^<}{e_1^> + e_2^<} \cdot e_1^> \cdot e_2^<}_{case\ 4} + \underbrace{e_1^< \cdot e_2^i \cdot d_2^<}_{case\ 5} + \\
& + \underbrace{\frac{e_1^<}{e_1^< + e_2^i \cdot d_2^=} \cdot e_1^< \cdot e_2^i \cdot d_2^=}_{case\ 6} + \underbrace{\frac{e_2^i \cdot d_2^<}{e_1^= + e_2^i \cdot d_2^<} \cdot e_1^= \cdot e_2^i \cdot d_2^<}_{case\ 7} + \underbrace{\frac{e_1^<}{e_1^< + e_2^i \cdot d_2^>} \cdot e_1^< \cdot e_2^i \cdot d_2^>}_{case\ 8} + \underbrace{\frac{e_2^<}{e_2^< + e_1^i \cdot d_1^>} \cdot e_1^i \cdot e_1^> \cdot d_2^<}_{case\ 9} + + \underbrace{e_1^i \cdot d_1^< \cdot e_2^<}_{case\ 10} + \\
& \underbrace{\frac{e_1^i \cdot d_1^<}{e_2^= + e_1^i \cdot d_1^<} \cdot e_1^i \cdot d_1^< \cdot e_2^=}_{case\ 11} + \underbrace{\frac{e_2^<}{e_2^< + e_1^i \cdot d_1^=} \cdot e_1^i \cdot e_1^= \cdot d_2^<}_{case\ 12} + \underbrace{\frac{e_1^i \cdot d_1^<}{e_2^> + e_1^i \cdot d_1^<} \cdot e_1^i \cdot d_1^< \cdot e_2^>}_{case\ 13} + \\
& + \underbrace{\frac{e_2^<}{e_2^< + e_1^i \cdot d_1^>} \cdot e_1^i \cdot e_1^> \cdot d_2^<}_{case\ 14} + \underbrace{e_1^i \cdot d_1^< \cdot e_2^i \cdot d_2^<}_{case\ 15} + \underbrace{\frac{e_1^i \cdot d_1^<}{e_1^i \cdot d_1^< + e_2^i \cdot d_2^=} \cdot e_1^i \cdot d_1^< \cdot e_2^i \cdot d_2^=}_{case\ 16} + \underbrace{\frac{e_2^i \cdot d_2^<}{e_1^i \cdot d_1^= + e_2^i \cdot d_2^<} \cdot e_1^i \cdot d_1^= \cdot e_2^i \cdot d_2^<}_{case\ 17} + \\
& \underbrace{\frac{e_1^i \cdot d_1^<}{e_1^i \cdot d_1^< + e_2^i \cdot d_2^>} \cdot e_1^i \cdot d_1^< \cdot e_2^i \cdot d_2^>}_{case\ 18} + \underbrace{\frac{e_2^i \cdot d_2^<}{e_1^i \cdot d_1^> + e_2^i \cdot d_2^<} \cdot e_1^i \cdot d_1^> \cdot e_2^i \cdot d_2^<}_{case\ 19}.
\end{aligned}
$$

Similarly we obtain expressions for $\Sigma^=$ and $\Sigma^>$. The total support of inconsistent relation e_r^i is equal to $\Sigma^< + \Sigma^= + \Sigma^>$ because it based on probabilities for all cases of inconsistency. Final values for $d_r^<, d_r^=, d_r^>$ are calculated using the same formulas as for composition operation.

Example 3 (continued). Now we can finish the example from Section 2. Let us remind that we have two vectors: $\left(\frac{1}{2}, \frac{1}{2}, 0, 0\right)$ and $\left(0, \frac{1}{2}, \frac{1}{2}, 0\right)$, that represent the same relation between two temporal points. We use the addition operation to combine them into one and with an accordance to the formulas from Definition 3.4 we have: $\left(\frac{1}{2}, \frac{1}{2}, 0, 0\right) + \left(0, \frac{1}{2}, \frac{1}{2}, 0\right) = \left(0, \frac{1}{4}, 0, \frac{3}{4}\left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]\right)$. It means, that: 1) with the probability $\frac{1}{4}$ the relation between ***a*** and ***b*** is “=”; with the probability $\frac{3}{4}$ it is inconsistent relation, and the percentage of each of the basic relations inside inconsistent one is equal to $\frac{1}{3}$.

Conclusion

Representation and reasoning with uncertain temporal relations are the main goals of this paper. We try to show one way to take into account values of all possible alternatives within one temporal relation as probabilities for basic relations. Also we consider the structure of possible inconsistency in temporal relation. The basic vector with seven parameters that represents a relation between two temporal points consists of two parts: inexact forth and inconsistent triad. The first part distributes probabilities among basic relations "<", ">", "=" and probability of inconsistency. The second part represents the composition for possible inconsistent relation: percentage of "<", of ">" and of "=" within the inconsistent relation. The reasoning mechanism allows to compose and find out addition of such temporal relations by recalculating values of vectors. Such representation makes it possible to evaluate final relation by providing exact measures for inexact and inconsistent parts.

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